**FEM Lab B**

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Here in this lab, the following equation is going to be solved.

With and and the homogeneous Dirichlet boundary conditions:

1. The weak formulation of this problem should be to find such that:

So, the FEM method is to find , such that

1. To compute the FEM, we discretize the domain into small small triangles. It is done by discretize each side of into equal sized pieces so it forms equally sized squares. Then each square is divided into triangles by connecting the upper-left corner and the lower-right corner. Thus, we shall get triangles. Thus inner nodes. We label these inner nodes from the lower-left corner toward right from to and continuous next row above it from to until . The lower-left part of the discretization is showed below:

II

III

I

3

2

1

VI

IV

V

Then, we choose that . Here is the node on the discretization. is the hat function . Since we are using the same discretization as we did in problem set A, we can use the same basis function as well.

From this, we can get the following stiffness matrix where:

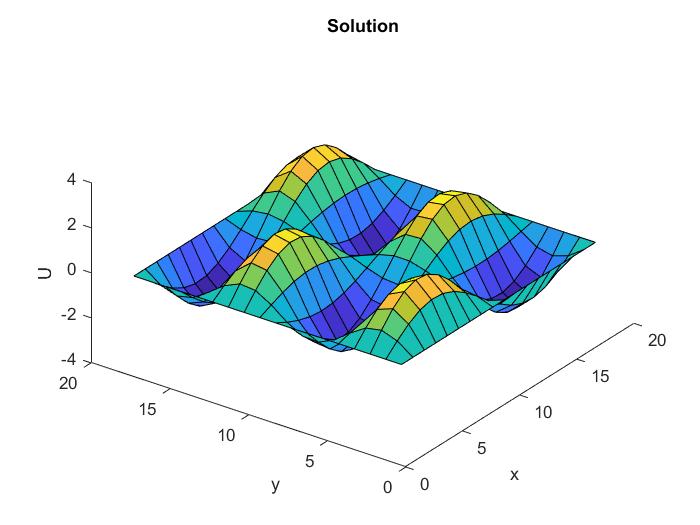
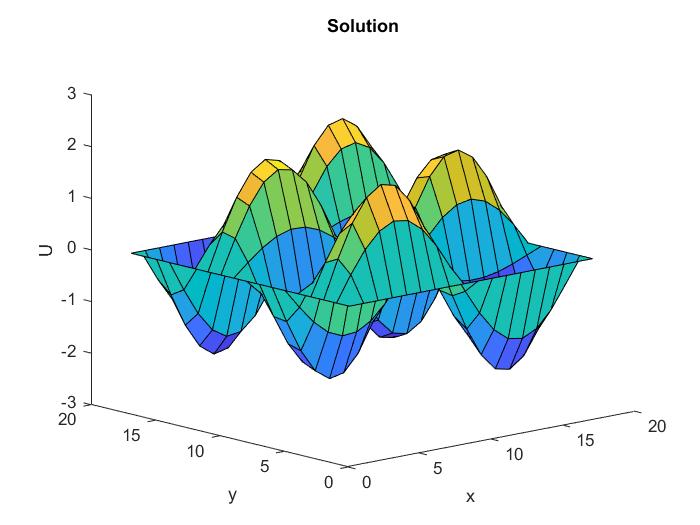
If , we can get the following values:

The load vector can also be calculated:

So, we shall have the FEM equation system:

Since the index goes from to , is a matrix and is a vector. The vector describe the solution of the entire range in a vector form. To compute the integration in the vector, the gaussian points integration is used because of its fair good accuracy.

1. Here we shall use the FEM above to compute the equation when and . The results are showed below in two directions.



Since we know the exact answer to the equation, the -norm of the error can be calculated by comparing the analytical result and the numerical result. The errors are showed below and we can see the FEM have a second order convergence in average.

|  |  |  |  |
| --- | --- | --- | --- |
| Step size |  |  |  |
| Error |  |  |  |

1. In this problem, we shall change the boundary condition on edges and from Dirichlet BC to Neumann BC:

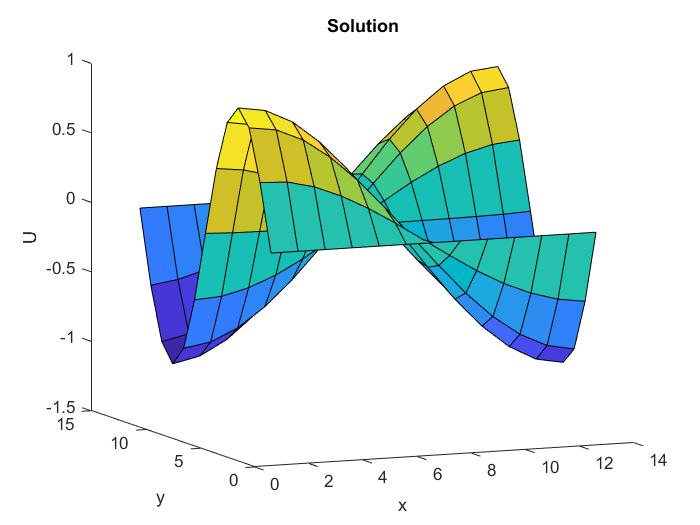
Beside this, the force function is changed to which corresponds to the exact solution . Then we can reformulate the weak formulation as to find such that:

Which gives the FEM:

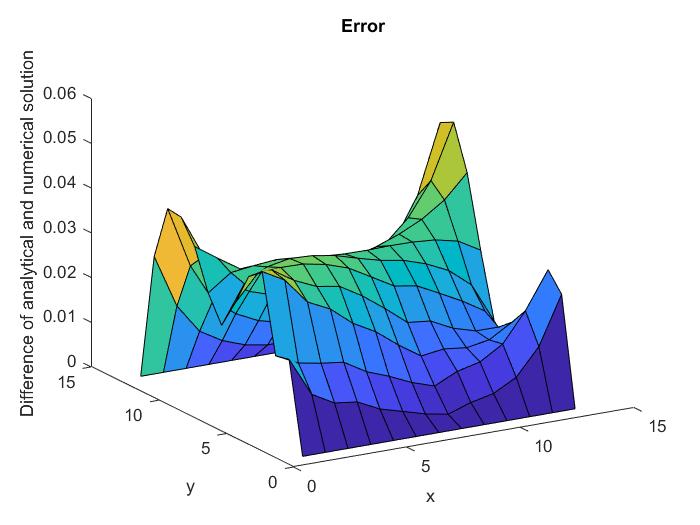
To find , such that:

The same assumption can be used. For the mesh, we add another two nodes on each row so their or . Then there are totally nodes. Let and . Then we get:

Which is a linear system with the form . The result can then be computed with MATLAB and is showed below.

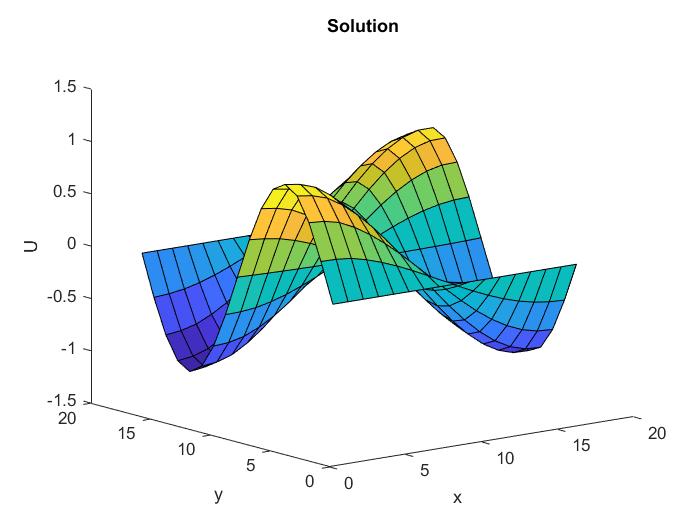


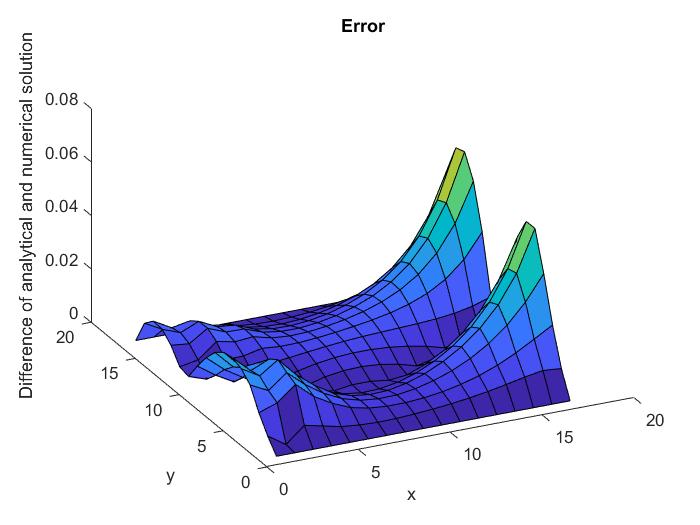
This solution is computed with . The -error corresponds to this is equal to . The error is plotted below.



We can see that the error is greatest near .

1. In this problem we change to . The plots of solution and error are showed below.





The – error is then 0.0182 which is smaller than before refinement. However the error is still big near